Math 4 Honors
Lesson 8-2: *Velocity & Net Change*

Name ___________________________________
Date __________________________________

Learning Goals:

- *I can use graphs to determine both rate and direction of change in a quantity.*
- *I can use directional rate graphs to calculate the net change in position of a moving object or the net flow in a pipeline.*

In work on the problems of Investigation 1, you developed strategies for using speed graphs to estimate distance traveled and rate graphs to estimate total flow through a pipeline. In each of the problems, the motion or flow was in one direction. But cars and people move both forward and backward, and pumps can be used to move fluids in both directions through a pipeline.

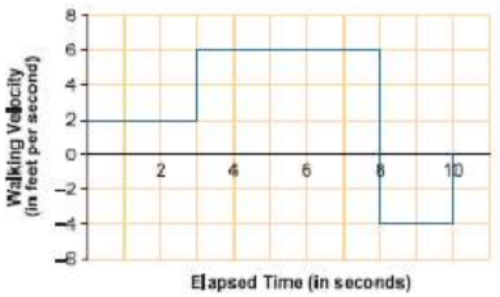
As you work on the problems of this investigation, look for answers to these questions:

How can graphs indicate both rate and direction of change in a quantity?

How can such directional rate graphs be used to calculate the net change in position of a moving object or the net flow in a pipeline?

Directed Motion To accurately describe the rate of change in position of a moving object, we need to specify both speed and direction of motion. The two attributes—speed and direction—define the term **velocity**. In situations involving motion toward and away from a sensor, it is customary to indicate speed in one direction with positive numbers and speed in the other direction with negative numbers.

1. Suppose that the following simplified graph shows the velocity of a person walking toward and away from a motion detector during a 10-second period.



Net Change: 28 ft

Total Change: 44 ft

a. How would you walk toward and away from the detector in order to replicate the given graph?

b. How far from the motion detector would you be after 10 seconds if you began your walk:

i. 5 feet from the detector?

ii. 3 feet from the detector?

iii. 12 feet from the detector?

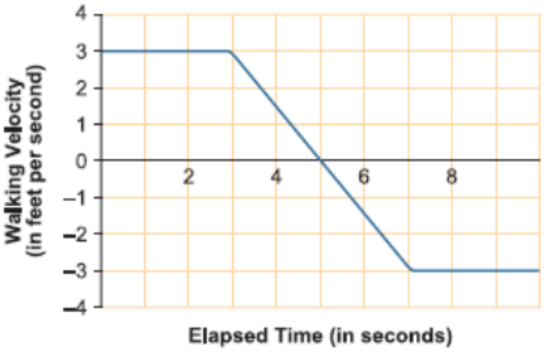
c. What is the net change in distance from the motion detector that results from each 10-second walk described in Part b?

d. What is the total distance you covered in each 10-second trip described in Part b?

OVER →

Page 2

2 Here is another graph showing the velocity of a person walking toward and away from a motion detector during a 10-second period.



a. How would you walk toward and away from the detector in order to replicate the given graph?

b. How far from the motion detector would you be after 10 seconds if you began your walk:

i. 6 feet from the detector?

ii. 4 feet from the detector?

iii. 9 feet from the detector?

c. What is the net change in distance from the motion detector that results from each 10-second walk described in Part b?

d. What is the total distance you covered in each 10-second trip described in Part b?

Net Change: 0 ft

Total Change: 24 ft

1 **a.** To produce the given graph, you would walk at a constant rate of 2 feet per second for 3 seconds away from the detector, then shift to a more rapid pace of 6 feet per second for the next 5 seconds, then shift to a pace of 4 feet per second toward the detector for 2 seconds.

b. **i.** 33 feet away

ii. 31 feet away

iii. 40 feet away

c. The net change in position is 28 feet in each case, a fact that students might have recognized before they even tackled Part b.

d. The total distance covered is 44 feet.

2 **a.** To walk the graph in this problem, you would want to start out walking at a constant speed of 3 feet per second away from the detector for 3 seconds, decrease speed to 0 feet per second at a constant rate over the next 2 seconds, reverse direction and increase speed at a constant rate over the next 2 seconds until you reach a speed of 3 feet per second in reverse, and then maintain that 3 feet per second backward pace for 3 more seconds.

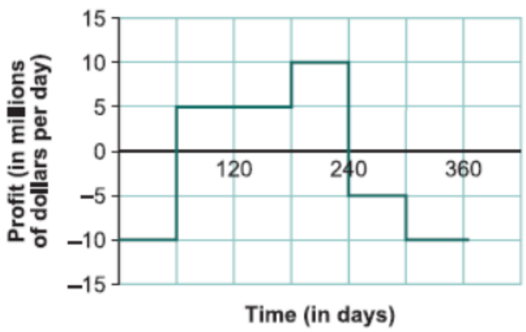
b. **i–iii.** Following the given rate graph, one always ends up where you started.

c. The net change in position is 0 feet.

d. The total distance covered is 24 feet.

4 For commercial airlines, one important variable factor in their profit picture is the price of jet fuel for their planes. When fuel prices rise, the airlines tend to lose money every day; when fuel prices fall, they tend to make money every day.

Suppose that the following graph shows variation in profit per day for an airline during one twelve-month period.



a. What pattern of change in the price of jet fuel is suggested by the profit graph?

b. What was the net profit of the airline:

- **i.** over the first 120 days of the year?
- **ii.** over the first 240 days of the year?
- **iii.** over the first 360 days of the year?

OVER →

Answers

4 a. The profit graph suggests high fuel prices for the first 60 days of the year, then lower prices for the next 120 days and even lower prices for the next 60 days. Then the fuel price must have risen during the next 60 days and risen even more during the final 60 days of the year.

b. Students' choice of time intervals may vary, but their method should be similar to the following: To calculate profit or loss over any time interval, multiply profit per day by number of days in the time interval—taking care to treat profit and loss periods separately before combining them.

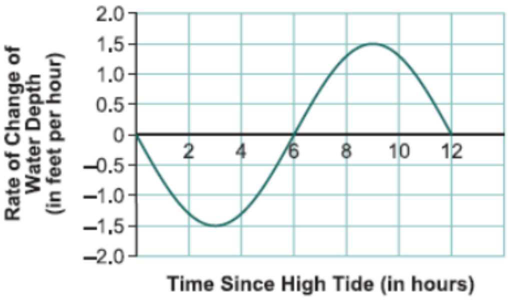
i. Loss of $300 million

ii. Profit of $600 million

iii. Loss of $300 million

Page 4

5 The next graph shows the rate at which water depth changes (in feet per hour) in the tidal water of an ocean harbor during one full period of the tide.



a. Explain what the pattern in the graph tells about change in water depth over each of these time intervals:

i. $0 \leq t \leq 3$ ii. $3 \leq t \leq 6$ iii. $6 \leq t \leq 9$ iv. $9 \leq t \leq 12$

b. Use the graph to find what you believe is an accurate approximation for the net change in water depth in the harbor over the first six hours of the period shown.

c. Use the symmetry of the graph and your result from Part b to estimate the net change in water depth over the time interval $6 \leq t \leq 12$.

d. What is the approximate net change in water depth over the time interval $0 \leq t \leq 12$?

Algebraic Maintenance!

31 Find all solutions to each equation.

a. $4 \sin \theta = -2$

b. $\tan^2 \theta + 2 \tan \theta + 1 = 0$

c. $\sqrt{x+4} = x - 2$

d. $1.3e^{x+2} = 123.5$

Answers

5 a. i. $0 \le t \le 3$; water depth decreasing at increasing absolute rate

ii. $3 \le t \le 6$; water depth decreasing at decreasing absolute rate

iii. $6 \le t \le 9$; water depth increasing at increasing rate

iv. $9 \le t \le 12$; water depth increasing at decreasing rate

b. Water depth appears to drop by about 6 feet in the first 6 hours. (At this stage, exact estimates are not as important as plausible reasoning and a number "in the ballpark.")

c. Between hours 6 and 12, the water level should rise about 6 feet.

d. Between 0 and 12 hours, the net change in water depth will be about 0 feet.